MATHEMATICS & STATISTICS - I

SOLUTION – COMPLEX NUMBERS

Q.1. Solve [Any 3] (2 Marks each) (06)

1) $z = 1 + 3i$, here $a = 1$, $b = 3$

$\therefore\ |z| = \sqrt{1^2 + 3^2} = \sqrt{10}$

$\text{amp}\, z = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{3}{1}\right)$

$= \tan^{-1} 3$

2) $(1 + 2i)(-2 + i)$

$= -2 + i - 4i + 2i^2$

$= -2 - 3i - 2$

$= -4 - 3i$

3) $(i^4 + 3i)a + (i - 1)b + 5i^3 = 0$

$\therefore\ [(i^2)^2 + 3i]a + bi - b + 5(-i) = 0$

$\therefore\ (1 + 3i)a + bi - b - 5i = 0$

$\therefore\ a + 3ai + bi - b = 0 + 5i$

$\therefore\ (a - b) + (3a + b)i = 0 + 5i$

Equating real and imaginary parts.

$a - b = 0$ (I) & $3a + b = 5$ … (2)

$3a + b = 5$ Adding

$+\ \underline{a - b = 0}$

$4a = 5$

$\therefore\ a = \frac{5}{4}$

Substituting $a = \frac{5}{4}$ in … (1)

$\frac{5}{4} - b = 0$

$\therefore\ -b = \frac{-5}{4}$

$\therefore\ b = \frac{5}{4}$

$\therefore\ a = \frac{5}{4}$, $b = \frac{5}{4}$

4) $z = \dfrac{2+i}{(3-i)(1+2i)}$

$= \dfrac{2+i}{3+6i-i-2i^2}$

$= \dfrac{2+i}{5+5i} \times \dfrac{5-5i}{5-5i}$

$= \dfrac{10-10i+5i-5i^2}{25-25i^2}$

$= \dfrac{15-5i}{50}$

$= \dfrac{3}{10} - \dfrac{1}{10}i$

Here $a = \dfrac{3}{10}, b = -\dfrac{1}{10}$

Q.2. Solve [Any 4] (3 Marks each) (12)

1) Let $\sqrt{6+8i}$ = a + ib, a, b $\in$ R

On squaring

$6 + 8i = (a + ib)^2$

$6 + 8i = a^2 - b^2 + 2abi$

Equating real and imaginary parts,

$6 = a^2 - b^2$ . . . (1)

$8 = 2ab$ . . . (2)

$\therefore a = \dfrac{4}{b}$

$6 = \left(\dfrac{4}{b}\right)^2 - b^2$

$6 = \dfrac{16}{b^2} - b^2$

$\therefore b^4 + 6b^2 - 16 = 0$

$(b^2 + 8)(b^2 - 2) = 0$

$b^2 = -8$ or $b^2 = 2$

Now b is a real number

$\therefore b^2 \neq -8$

$\therefore b^2 = 2$

$\therefore b = \pm\sqrt{2}$

when $b = \sqrt{2}$, $a = 2\sqrt{2}$

$\therefore$ Square root of 6 + 8i

$= 2\sqrt{2} + \sqrt{2}\, i = \sqrt{2}(2 + i)$

when $b = -\sqrt{2}$, $a = -2\sqrt{2}$

$\therefore$ Square root of 6 + 8i

$= -2\sqrt{2} - \sqrt{2}\, i = -\sqrt{2}(2 + i)$

$\sqrt{6+8i} = \pm\sqrt{2}(2 + i)$

2) $\left[\frac{\sqrt{3}}{2}+\frac{i}{2}\right]^3=\left[\frac{\sqrt{3}+i}{2}\right]^3$

$= \frac{(\sqrt{3})^3+3(\sqrt{3})^2 i+3(\sqrt{3})(i)^2+i^3}{8}$

$= \frac{3\sqrt{3}+9i-3\sqrt{3}-i}{8}$

$= \frac{8i}{8}$

$= i$

$\therefore$ L.H.S.

= R.H.S.

3) $\frac{a+3i}{2+ib} = 1 - i$

$\therefore a + 3i = (2 + ib)(1 - i)$

$\therefore a + 3i = 2 - 2i + bi - bi^2$

$\therefore a + 3i = 2 - 2i + bi + b$ . . . $[\because i^2 = -1]$

$\therefore a + 3i = (b + 2)(b - 2)i$

Equating the real and imaginary parts separately, we get,

a = b + 2 and 3 = b - 2

a = b + 2 and b = 5

$\therefore a = 5 + 2 = 7$

$\therefore 5a - 7b = 5(7) - 7(5)$

$= 35 - 35$

$= 0$

4) Find the values of

(i) $z = i^{49} + i^{68} + i^{89} + i^{110}$

$= (i^4)^{12}(i) + (i^4)^{17} + (i^4)^{22}(i) + (i^4)^{27}(i^2)$

$= (1)^{12}(i) + (1)^{17} + (1)^{22}(i) + (1)^{27}(-1)$

$= i + 1 + i - 1$

$= 2i$

(ii) $z = i + i^2 + i^3 + i^4$

$= i - 1 - i + 1$

$= 0$

5) $z = 1 + i^{10} + i^{20} + i^{30}$

$= 1 + (i^4)^2(i^2) + (i^4)^5 + (i^4)^7(i^2)$

$= 1 + (1)^2(-1) + (1)^5 + (1)^7(-1)$

$= 1 - 1 + 1 - 1$

$= 0$ which is a real number.

6) $(a + bi)(1 - i) = 1 + i$

$a - ai + bi - bi^2 = 1 + i$

$(a + b) + (b - a)\, I = 1 + i$

Comparing coefficients we get

$a + b = 1$ …… (1)

$b - 1 = 1$ …… (2)

Add (1) + (2)

$a + b = 1$

$-\ a + b = 1$

$2b = 2$

$b = 2$

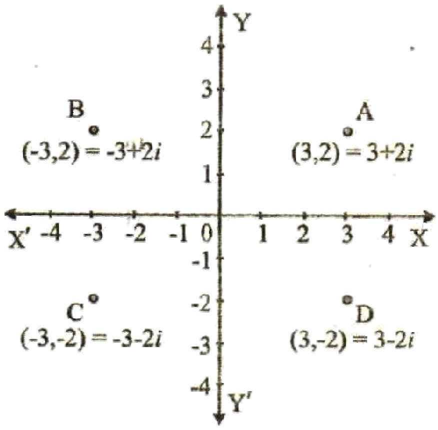
$\therefore\ a = 0$

$a^2 + b^2 = (0)^2 + (1)^2$

$a^2 + b^2 = 1$

Q.3. Solve [Any 3] (4 Marks each) (12)

1) The complex number 3 + 2i, 3 – 2i, – 3 + 2i, – 3 – 2i will be represented by the points (3, 2), (3, – 2), (–3, 2), (–3, – 2) as shown in the following figure.



2) Let $z = -1 + \sqrt{3}i$

On comparing with a + bi, a = -1 b = $\sqrt{3}$

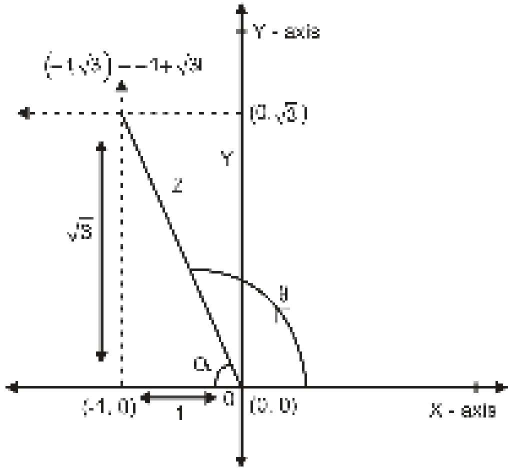
$\therefore\ r = \sqrt{a^2 + b^2}$

$= \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2}$

$= \sqrt{1 + 3}$

$= \sqrt{4}$

$= 2$



$\cos\ a = \frac{1}{2}$

But $\cos 60^0 = \frac{1}{2}$

\ $\cos\ a = \cos 60^0$

\ $a = 60^0$

$a + q = 180^0$ [Linear pair]

\ $60^0 + q = 180^0$

\ $q = 120^0$

The polar form of complex number $-1 + \sqrt{3}i$ is $r(\cos q + i \sin q)$

$= 2(\cos 120^0 + i \times \sin 120^0)$

$= 2\left[\cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3}\right]$

3) Since $x = 1 + \sqrt{3}i$

$\therefore \quad (x - 1) = \sqrt{3}i$

squaring

$(x - 1)^2 = (\sqrt{3}i)^2$

$x^2 - 2x + 1 = 3i^2$

$x^2 - 2x + 1 = -3$

$\therefore \quad x^2 - 2x + 4 = 0$

$$\begin{array}{r|l} & x+1 \\ x^2 - 2x + 4 & x^3 - x^2 + 2x + 4 \\ & x^3 - 2x^2 + 4x \\ & (-)\ (+)\ (-) \\ \hline & x^2 - 2x + 4 \\ & x^2 - 2x + 4 \\ & (-)\ (+)\ (-) \\ \hline & 0 \end{array}$$

$x^3 - x^2 + 2x + 4 = (x + 1)(x^2 - 2x + 4) + 0$

$= (x + 1)(0)$

$x^3 - x^2 + 2x + 4 = 0$

4) Let $\sqrt{-16 + 30i} = x + yi$, where $x, y \in R$

On squaring both sides, we get

$-16 + 30i = (x + yi)^2 = x^2 + y^2i^2 + 2xyi$

$\therefore -16 + 30i = (x^2 - y^2) + 2xyi \quad \ldots [\because i^2 = -1]$

Equating real and imaginary parts separately, we get,

$x^2 - y^2 = -16$ and $2xy = 30$

$\therefore y = \dfrac{15}{x}$

$\therefore x^2 - \left(\dfrac{15}{x}\right)^2 = -16$

$\therefore x^2 - \dfrac{225}{x^2} = -16$

$\therefore x^4 - 225 = -16x^2$

$\therefore x^4 + 16x^2 - 225 = 0$

$\therefore (x^2 + 25)(x^2 - 9) = 0$

$\therefore x^2 = -25$ or $x^2 = 9$

Now x is a real number

$\therefore x^2 \neq -25$

$\therefore x^2 = 9$

$\therefore x = \pm 3$

When $x = 3$, $y = \dfrac{15}{3} = 5$

When $x = -3$, $y = \dfrac{15}{-3} = -5$

$\therefore$ the square roots of $-16 + 30i$ are $3 + 5i$ and $-3 - 5i$

i.e., $\pm(3 + 5i)$